A Trivial Deterministic Hidden Variable Model for Quantum Circuits

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I. Introduction

Few realist models in the academic literature are well-suited for analyzing quantum circuits. These models often rely on physical parameters that lack direct relevance to the abstract framework of quantum information theory, limiting their utility in practical applications. The purpose of the model presented here is to address this gap by constructing a hidden variable framework that operates entirely within the abstract formalism of quantum information. This approach ensures compatibility with the analysis of quantum circuits, regardless of the specific physical implementation.

The key insight lies in leveraging the fact that the global phase of a quantum state is inherently unobservable. By treating this phase as a random variable, we can incorporate it into the model without violating the principles of quantum mechanics. When a quantum circuit is updated, the unaffected qubits are first conditioned upon, allowing the probability distribution derived from the Born rule to be tied to a specific configuration of the system. A cumulative distribution function (CDF) is then used to map this probabilistic structure to a deterministic update rule for the affected qubits. This ensures that the model remains consistent with the stochastic nature of quantum measurements while preserving the deterministic evolution of the system's underlying structure.

By framing the global phase as a source of variability and using the CDF to mediate the transition between probabilistic and deterministic descriptions, the model provides a novel framework for interpreting quantum dynamics. This approach not only aligns with the mathematical foundations of quantum theory but also offers a practical tool for analyzing the behavior of quantum circuits in a realist, hidden-variable context.

II. A Global Model

A. Computational Basis

The statistical distribution of qubit measurement outcomes in a quantum circuit at a specific moment is determined by the Born rule,[1] as defined in Equation (1).

$$Pr(x|\psi) = |\langle x|\psi\rangle|^2 \tag{1}$$

In a realist framework, qubits must be considered to possess an ontic state. Consequently, the probability distribution derived from Equation (1) should be interpreted as a ψ-epistemic model, reflecting the observer's incomplete knowledge of the system's true state rather than indefiniteness. The quantum computer's memory is assumed to have a definite configuration in the physical world, with the statistical distribution merely encoding the observer's ignorance about its exact state.

A quantum circuit can be represented in an arbitrary basis. However, interpreting the probability distribution epistemically poses a challenge: it would imply that the quantum system's ontic state exists simultaneously across all possible bases, directly contradicting the Kochen-Specker theorem.[2]

This issue can be resolved by designating the computational basis as the privileged reference frame. The probability distribution in Equation (1) then accurately represents an epistemic description of the system only when the quantum state is expressed in the computational basis.

Any arbitrary quantum circuit measured in a non-computational basis can be equivalently described in the computational basis through a change of representation. This is analogous to the act of rotating a globe: if one walks around a stationary globe at an angle θ, or rotates the globe itself by -θ while remaining stationary, the perceived orientation of features on the globe's surface remains identical.

Thus, a rotation of the measurement apparatus can be recast as an inverse rotation of the qubit's quantum state. This inverse transformation corresponds to the time-reversed process, represented mathematically by the Hermitian transpose of the unitary operator describing the measurement device's rotation.

The operator describing a rotation of the measurement apparatus is given by Equation (2). To determine the unitary transformation U, we assume the computational basis is represented by Z. Equation (3) provides the unitary transformation required to rotate the measurement device for measuring in the X-basis, while Equation (4) specifies the corresponding transformation for the Y-basis.

$$U\mathcal{O}U^\dagger = Z \tag{2}$$

$$HXH^\dagger = Z \tag{3}$$

$$(SH)Y(SH)^\dagger = Z \tag{4}$$

By applying the Hermitian transpose of the Hadamard gate as defined in Equation (3), we can equivalently perform measurements in the X-basis by instead measuring in the Z-basis. Similarly, measurements in the Y-basis can be recast as Z-basis measurements by applying the Hermitian transpose of the product of the Phase-S and Hadamard gates, as derived in Equation (4). This approach enables all orthogonal measurements to be expressed in the same computational basis.

Since measurements that are not aligned with the computational basis require the application of unitary operators prior to measurement, such operations cannot be interpreted as passive observations that merely reveal the system's ontic state to the observer. Instead, these measurements must be understood as active transformations of the ontic state, reflecting the system's behavior under external manipulation rather than direct access to its ontic configuration. Only measurements performed in the computational basis can be considered passive revelations of the quantum system's ontic state, as they do not necessitate prior unitary intervention.

This privileging of a particular basis and interpreting the measurements misaligned with the privileged basis as emergent properties of the system is not original to us. It is similar to a proposal by Jacob Barandes.[3]

B. Global Phase Factor

The global phase of a quantum state does not influence the statistical outcomes of measurements given by Equation (1), as it remains invariant under unitary transformations.[4] This invariance allows us to define the initial configuration of a quantum circuit, as shown in Equation (5), where $|x\rangle$ represents an arbitrary quantum state and λ denotes the global phase.

$$|\psi\rangle = |x\rangle e^{i\lambda} \tag{5}$$

Due to the fact that that the global phase does not alter the empirical statistics, it is typically interpreted to not be a physical property of the system. For the purpose of this toy model, it is convenient to treat it as if it is a real property of the system. This choice, used in this model, should not be understood to have any physical implications, but is merely a choice of convenience, as it allows us to utilize a free variable already baked into the structure of quantum information.

If the global phase λ does not influence the system's physical properties and is therefore empirically unobservable, it is irrational to assume that λ necessarily has a definite value at the beginning of the quantum circuit. Consequently, it is more reasonable to treat λ as a random variable. This implies that the global phase is randomly selected rather than predetermined.

While the statistics in Equation (1) remain unaffected by variations in λ, the phases of the quantum state described by Equation (6) can vary significantly at each time step during the computation when λ is varied.

$$\phi = \arg(\psi) \tag{6}$$

This enables the definition of γ, as given in Equation (7). γ is a parameter that evolves throughout the quantum circuit, encapsulating the cumulative phase offset resulting from variations in λ. If λ is uniformly distributed, γ will also follow a uniform distribution over the interval [0,1].

$$\gamma = \frac{\sum_k \phi_k \mod 2\pi}{2\pi} \tag{7}$$

This will be referred to as the global phase factor.

C. Discrete Update Rule

The quantum state evolves discretely over time according to a series of unitary operators, as defined in Equation (8). This model assumes a deterministic evolution of the quantum state throughout the computation.

$$|\psi'\rangle = U|\psi\rangle \tag{8}$$

Once the quantum state is updated, the ontic states of the qubits must also be updated. When the square amplitude of the unitary operator corresponds to a permutation matrix, the ontic state is updated directly via the permutation matrix, as shown in Equation (9).

$$\vec{s}\,' = \mathcal{P}\vec{s} \tag{9}$$

If the unitary operator does not correspond to a permutation matrix, a stochastic update rule must be applied. First, compute the joint probability distribution across the full system memory using Equation (1). Next, account for the unaffected qubits by conditioning the distribution on their known values. States conflicting with these fixed values are assigned zero probability, and the remaining probabilities are normalized by dividing by the total sum of the distribution, as shown in Equation (10).

$$Pr'(\mathbf{x}_A) = \frac{Pr(\mathbf{x}_A, \mathbf{x}_F = \mathbf{v}_F)\cdot \mathbb{I}(\mathbf{x}_F = \mathbf{v}_F)}{\sum_{\mathbf{y}_A} P(\mathbf{y}_A, \mathbf{x}_F = \mathbf{v}_F)} \tag{10}$$

This process is equivalent to a Bayesian update, where the prior global distribution is conditioned on the known values of the idle qubits. Equation (10) produces a statistical distribution that is non-degenerate for the qubits affected by the unitary operator and degenerate for those not affected. The next step is to define a cumulative distribution function (CDF) for this probability distribution, as shown in Equation (11).

$$F(x_k) = \sum_{i=1}^{k} Pr'(\mathbf{x}_A)_i \text{ where } F(x_0) = 0 \tag{11}$$

The CDF defined in Equation (11) can then be used to select a new configuration for the system, as shown in Equation (12), using the global phase factor γ.

$$\vec{s}\,' = x_k \iff F(x_{k-1}) \le \gamma < F(x_k) \tag{12}$$

An intuitive way to conceive of the CDF is an analogy to an analog clock with lines on the clock to separate its face into 2^N different zones. The size of each zone will be proportional to the likelihood of the associated configuration of bits occurring after conditioning on the idle bits. The value γ is the time on the clock and thus determines which zone the clock hand falls into and thus the system's specific configuration of 2^N possible configurations.

This configuration can then be used to construct a new permutation matrix. This matrix will be an identity matrix except for swaps between the new configuration given in Equation (12) and the current configuration. The methodology to construct this permutation matrix is detailed in Equation (13).

$$\mathcal{P} = \mathbf{I} - \vec{s}\vec{s}^{\top} - \vec{s}'\vec{s}'^{\top} + \vec{s}\vec{s}'^{\top} + \vec{s}'\vec{s}^{\top} \qquad (13)$$

Finally, this permutation matrix can be used to discretely evolve the qubits according to Equation (9). The choice of permutation matrix is deterministic, as it is ultimately determined by γ.

D. Continuous Update Rule

Equation (14) defines the continuous update rule. The time parameter t can be varied continuously from 0 to 1, allowing the quantum state to be computed at any intermediate point. The unitary transformation preserves probability throughout this evolution.

$$|\psi(t)\rangle = U^t|\psi(0)\rangle \qquad (14)$$

Once a permutation matrix is obtained for use in Equation (9), the continuous dynamics of the ontic states of the qubits can be described using unistochastic dynamics, as defined in Equation (15). Unistochastic dynamics refer to a process where the evolution of the system is governed by the square of a unitary matrix, allowing for a probabilistic interpretation of the state transitions.

$$\vec{p}(t) = |\mathcal{P}^t|^2\vec{s}(0) \qquad (15)$$

Equation (15) generates a vector representing a probability distribution. A new configuration of the system can be selected from this distribution using the method outlined in Equations (11) and (12), incorporating the global phase factor γ. The resulting dynamics cause the qubit values to oscillate during continuous updates before eventually converging to a stable configuration.

Another way to construct the continuous update rule is to return to the clock analogy. If γ represents the time on the clock face, and thus the angle of the clock hand, then one can continuously evolve γ (before the unitary operator is applied) to γ' (after the unitary operator is applied). The bits will change as the hand crosses over the different zones.

III. Discussion

We justify treating the global phase as a random variable due to its immeasurability and its lack of influence on the stochastic dynamics of the quantum circuit. This makes it rational to assume its value varies between experiments as a random variable rather than assigning it a fixed value.

There is, however, another justification for this choice. In our stochastic network interpretation of quantum information,[5] the quantum state is decomposed into two real-valued vectors: one corresponding to the probability distribution given by Equation (1), and the other representing the phases described by Equation (6). These vectors are updated directly without reverting to the traditional quantum mechanical formalism.

This approach eliminates the need for the collapse of the wavefunction, as only the probability vector must be updated at the moment of measurement. This corresponds to a Bayesian knowledge update on the probability vector. The phase vector, interpreted as a deterministic property of the system, remains unaffected by the measurement process. Consequently, it is not meaningful to update the phase vector through a knowledge update.

If the phases are not updated, the measurement rule does not require stripping away the phase factors. Instead of reducing the quantum state to a basis state without a phase factor, the measurement rule can be defined in a similar way as Equation (10) where we zero out probability amplitudes for any state conflicting with these fixed values and normalize the remaining quantum state.

This rule ensures that incompatible observations are set to zero, and the quantum state is normalized accordingly. It preserves the integrity of the phase vector while updating the probability distribution.

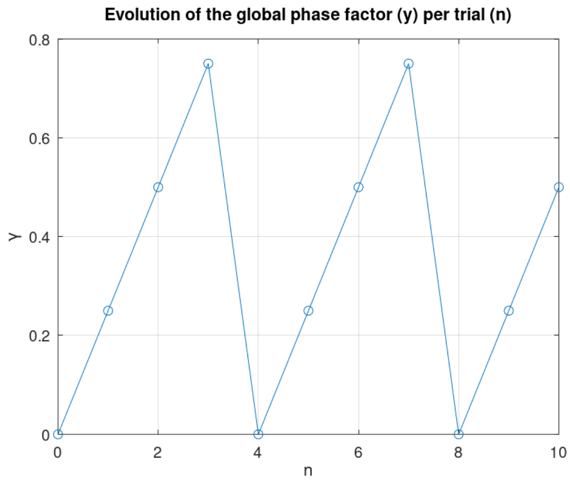
Consider a quantum circuit analogous to the Mach-Zehnder interferometer, where a single measurement is performed on the outputs of the two beam splitters. The beam splitter operator is defined in Equation (16).

$$B = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad (16)$$

Now, consider repeating the experiment with the same qubit. We assume the qubit's state is reversed using the Pauli-X gate to restore it to its original configuration. At each iteration, a measurement is performed using a phase-preserving update rule. This process resets the qubit to an eigenstate while preserving its phase factor. As a result, the entire circuit can be represented using Equation (17), where n denotes the number of trials.

$$|\psi\rangle = (XBB)^n \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{17}$$

When we graph how γ evolves over trials, it becomes clear that γ is not consistent across trials.



Thus, we can reasonably conclude that λ, and by extension γ, does not always begin with a definite value at the start of each quantum circuit. Instead, it must be treated as a random variable. This probabilistic treatment enables λ to play a deterministic role within this model.

References

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