A Stochastic Network Interpretation of Quantum Information

pcalau12i@foleosoft.com

March 8, 2026

I. Introduction

We present a formalism for quantum information that reinterprets the two degrees of freedom in the quantum state as representing an epistemic statistical distribution over the configuration of bits in a quantum computer's memory and an ontic phase network. The phase network is posited as a deterministic, relational property of the system, encoding its internal structure and influencing the stochastic perturbations of the bits during quantum gate operations. Our approach replaces the collapse of the wavefunction with Bayesian inference, describing measurement updates as probabilistic updates to the observer's knowledge, rather than as a physical collapse of the system's state. The phase network, analogous to the parameters of a neural network, evolves independently of the stochastic configuration of the bits, maintaining its ontic role as a relational property of the system. The quantum state is then understood as a coordinate transformation from polar to Cartesian coordinates, serving as a computational convenience. The Born rule is then understood to be a result of this computationally convenient encoding and not a foundational postulate.

II. The Formalism

A. Splitting the Quantum State

The quantum state is represented by a complex-valued vector ψ. Complex numbers inherently encode two degrees of freedom, which can be analyzed by decomposing the system into two real-valued representations. This approach facilitates a clearer understanding of the physical significance of these degrees of freedom.

To begin, we decompose the quantum state into two real-valued vectors. Since complex numbers are conventionally interpreted as Cartesian coordinates on the complex plane, we can express the quantum state as a pair of real-valued coordinates, as shown in Equations (1) and (2).

$$\vec{x} = \Re(\psi) \tag{1}$$

$$\vec{y} = \Im(\psi) \tag{2}$$

If these vectors are interpreted as Cartesian coordinates, they can be transformed into polar coordinates without losing information about their two degrees of freedom. This transformation yields two new real-valued vectors, as defined in Equations (3) and (4).

$$\vec{p} = \vec{x} \odot \vec{x} + \vec{y} \odot \vec{y} \tag{3}$$

$$\phi = \operatorname{arctan2}(\vec{y}, \vec{x}) \tag{4}$$

The derivation of these vectors in terms of ψ is detailed in Equations (5) and (6).

$$\vec{p} = |\psi|^2 \tag{5}$$

$$\phi = arg(\psi) \tag{6}$$

We propose that the quantum state should be interpreted as encoding two distinct degrees of freedom: the probability vector (Equation 3) and the phase vector (Equation 4). The probability vector is treated as epistemic, reflecting uncertainty about the system's state. In quantum computing, qubits are merely bits that exist in definite configurations, but the dynamics of quantum circuits are fundamentally stochastic. This limits our ability to track the ontic states of individual qubits, instead requiring probabilistic descriptions as given in Equation (3).

The interpretation of the statistical distribution as epistemic uncertainty is valid only in the computational basis. This aligns with Barandes' framework,[1] which advocates privileging a specific basis for representation. If we were to interpret the statistical distribution as epistemic uncertainty across all bases simultaneously, it would imply the system has definite bit values for every possible basis. This contradicts the Kochen-Specker theorem,[2] necessitating the selection of a privileged basis.

The phase vector in Equation (4) is interpreted as an ontic property of the system. It does not represent a substantive entity but rather a relational characteristic of the system, as discussed in greater detail later.

B. Update Rules

Consider a probabilistic computer where the memory state is represented by a vector of probabilities. Each logic gate is then described by a stochastic matrix that governs the evolution of this probability vector. The system's update rule is thus given by Equation (7), where Γ denotes the stochastic matrix.

$$\vec{p}' = \Gamma \vec{p} \tag{7}$$

Equation (7) can alternatively be expressed as an element-wise summation, as shown in Equation (8).

$$\vec{p}_j' = \sum_k \Gamma_{jk} \vec{p}_k \tag{8}$$

While this formulation captures the behavior of a probabilistic computer, it is insufficient to describe the probabilistic nature of quantum computation. To extend this framework to quantum systems, we incorporate the phase vector ϕ, which accounts for the system's relational dynamics. This is formalized in Equation (9).

$$\vec{p}_j' = \sum_k \Gamma_{jk} \vec{p}_k + f(\phi) \tag{9}$$

The phase vector ϕ is interpreted as a deterministically evolving property of the system. When the memory bits of a quantum computer undergo stochastic perturbations, the exact nature of these perturbations is influenced by the current state of the phase vector. The mechanism by which this occurs is defined in Equation (10).

$$f(x) = \sum_{k<l} 2\sqrt{\Gamma_{jk}\Gamma_{jl}\vec{p}_k\vec{p}_l}\cos(x_k - x_l + \Theta_{jk} - \Theta_{jl}) \quad (10)$$

While Γ represents a stochastic matrix, Θ denotes an additional real-valued matrix referred to as the phase matrix. The derivation of these matrices from a unitary operator is detailed in Equations (11) and (12), respectively.

$$\Gamma = |U|^2 \quad (11)$$

$$\Theta = \arg(U) \quad (12)$$

To complete the formalism, we must also define update rules for the phase vector. This representation relies on polar coordinates, where the phase vector encodes angular information. To compute these angles, we require Cartesian coordinates, which are derived from the probability vector and phase vector via the function defined in Equation (13).

$$g(j, \epsilon) = \sum_k \sqrt{\Gamma_{jk}\vec{p}_k}\sin\Big(\phi_k + \Theta_{jk} + \epsilon\Big) \quad (13)$$

Using this function, the update rule for the phase vector is given in Equation (14).

$$\phi'_j = \text{arctan2}\left(g(j,0), g(j, \frac{\pi}{2})\right) \quad (14)$$

Thus, the probability and phase vectors can be updated directly based on the description of quantum logic gates in terms of stochastic and phase matrices, eliminating the need to revert to the orthodox formalism of complex-valued quantum states represented by ψ.

C. Measurement

Consider a quantum algorithm in which the qubits in the quantum computer are not in a known state. The observer possesses only a probability distribution over the ontic states of the memory. Now, suppose the observer measures a single qubit of the quantum computer's memory, but not all of them.

Since the probability vector represents a classical distribution of probabilities, the observer can update it using Bayes' theorem,[3] as formalized in Equation (15).

$$Pr(H_k|E) = \frac{Pr(E|H_k)Pr(H_k)}{Pr(E)} \quad (15)$$

To illustrate, consider a quantum computer with two qubits. After executing a specific quantum circuit, the observer assigns a probability vector to the memory, as shown below.

$$\vec{p} = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{pmatrix}$$

The number of hypotheses about the memory state equals 2^N, where N is the number of qubits. For N=2, there are 2^2=4 hypotheses: H_{00}, H_{01}, H_{10}, and H_{11}, corresponding to all possible configurations of the memory.

Now, suppose the observer measures the most significant bit and finds it to be in state 1. To compute the probability of this evidence, we use the general definition provided in Equation (16).

$$Pr(E) = \sum_k (Pr(E|H_k)Pr(H_k)) \quad (16)$$

The prior probability of a hypothesis $Pr(H_k)$ is simply the value assigned to it in the probability vector. The likelihood $Pr(E|H_k)$ is 100% if the evidence E is compatible with H_k, and 0% otherwise, as incompatible evidence cannot occur under that hypothesis.

If the most significant bit is measured to be 1, the updated probabilities are as follows: the first two hypotheses (H_{00}, H_{01}) are incompatible with the evidence and are thus reduced to 0%.

$$Pr(E|H_{00}) = 0$$
$$Pr(E|H_{01}) = 0$$
$$Pr(E|H_{10}) = 1$$
$$Pr(E|H_{11}) = 1$$

With this information, we can apply Bayes' theorem (Equation 15) to compute the updated probability vector, as shown below.

$$\vec{p} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.4286 \\ 0.5714 \end{pmatrix}$$

This process is known as Bayesian inference. As a further example, if the least significant bit is measured and found to be 0, a similar update can be performed, yielding the new probability vector given below.

$$\vec{p} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

The key insight of this formulation is that Bayesian inference suffices to account for measurement updates. The phase vector, which evolves deterministically through the system, does not require updating upon measurement. Unlike the probability vector, the phase vector does not represent a statistical distribution, and thus Bayes' theorem is not applicable to it.

D. Information-Sharing Principle

Consider a quantum circuit involving two qubits initialized in the $|00\rangle$ state. A Hadamard operator is applied twice to the most significant qubit. The resulting circuit and its corresponding probability vector are illustrated below. Here, bit vectors denoted by a lower-case B represent real-valued basis vectors corresponding to subscript.

$$\vec{p} = |(H \otimes I)(H \otimes I)|^2 \vec{b}_{00} = \vec{b}_{00}$$

Now, consider a second quantum circuit where the least significant qubit is not left idle. Instead, the state of the most significant qubit is recorded onto the least significant qubit using the CNOT operator.

$$\vec{p} = |(\mathrm{H} \otimes \mathrm{I})\mathrm{CNOT}(\mathrm{H} \otimes \mathrm{I})|^2 \vec{b}_{00} = 0.5(\vec{b}_{00} + \vec{b}_{11})$$

In the first case, the marginal probabilities for the most significant qubit follow a degenerate distribution, as the repeated Hadamard operations collapse the state to a classical bit. In contrast, the second circuit produces a uniform distribution for the most significant qubit. This difference is significant: the CNOT operator is a passive, non-interacting transformation that merely copies the control qubit's state to the target qubit.

The control qubit (most significant qubit) should remain unaffected by the CNOT operation, yet its marginal probabilities are altered by the introduction of the CNOT gate. This phenomenon, which we term the Information-Sharing Principle (ISP), arises from the correlation established between the two qubits. Even though the CNOT operation is passive in the moment, it creates a dependency between the qubits, which can influence the statistical outcomes of future logic gates. This is a direct consequence of the update rules formalized in Equations (9) and (14).

The ISP underscores a fundamental principle: knowing something about a quantum system's state requires physical interaction. If an observer gains epistemic knowledge of a qubit's state, this knowledge must be encoded through a physical correlation such as an interaction between the qubit and a measuring device. While such a measurement does not immediately alter the qubit's state, it introduces a statistical dependency that can affect future measurements. This aligns with the idea that quantum mechanics inherently links information and physical interactions, even in seemingly passive operations.

III. Discussion

A. Primacy of Our Formalism

If a textbook were to present quantum mechanics using this formalism, where each unitary operator is expressed as a combination of a stochastic matrix and a phase matrix, then, guided by these update rules, it would be possible to evolve the probability vector and phase vector directly. This would eliminate the need to revert to the traditional formalism of quantum mechanics, which relies on the complex-valued quantum state ψ.

Therefore, this formalism should not be viewed as a derivative or secondary representation of the orthodox framework. Instead, it offers a self-contained, independent description of quantum dynamics. It is mathematically equivalent to the orthodox formalism but does not presuppose it. One could equally argue that the orthodox formalism presupposes this one, as the latter provides a foundational interpretation of quantum states, while the former serves as a more convenient mathematical shorthand for calculations.

The precedence of the orthodox formalism historically does not justify its dominance in interpretative ontology. The formalism presented here should instead be granted interpretive primacy, with the orthodox framework regarded as a computational tool that simplifies expressions and calculations.

The probability vector and phase vector emerge from a coordinate transformation from Cartesian to polar coordinates. This transformation does not introduce new elements to the formalism but re-expresses the two inherent degrees of freedom already present in the quantum state ψ. Thus, even within the orthodox formalism, ψ can be interpreted as a simultaneous representation of the probability vector and phase vector, rather than the reverse.

B. The Born Rule

The equivalence between the definition of the probability vector in terms of the quantum state (Equation 5) and the Born rule[4] is of interest. The probability vector is thus guaranteed to always reflect the same statistical outcomes as the squared magnitude of the wave function, rendering the Born rule redundant as an independent postulate.

In our interpretation, the primary mathematical objects are the probability vector and the phase vector. The probability vector is inherently probabilistic, always encoding the statistical distribution of the current configuration of the quantum computer's memory. This ensures that the quantum system's behavior is consistently described by stochastic dynamics, even as the phase vector evolves deterministically as an ontic state of the system.

The Born rule, in this framework, emerges as a coordinate transformation from polar to Cartesian coordinates. This transformation does not introduce new physical content but re-expresses the two degrees of freedom (probability and phase) in a form that simplifies calculations. However, this representation obscures the direct physical meaning of the probability vector and phase vector, as their roles become less explicit in the complex-valued formalism.

The quantum state is not interpreted as a singular, collapsing entity upon measurement. Instead, the bits in the quantum computer's memory are always in a definite configuration, as represented by the probability vector. This configuration undergoes stochastic perturbations, which are influenced by the current state of the phase vector. The phase vector, evolving deterministically, acts as a relational, ontic property of the system, distinct from the epistemic uncertainty captured by the probability vector.

Crucially, we do not posit any physical collapse of the quantum state upon measurement. Instead, the observer performs a Bayesian inference on the evidence obtained from the measurement. While this process influences the future statistics of the bits in subsequent interactions (via the ISP), it does not alter the system's state at the moment of measurement. The measurement updates the probability vector, reflecting new knowledge about the system's configuration, but the phase vector remains unchanged as a deterministic ontic state.

Thus, the orthodox formalism of quantum mechanics, with its reliance on the complex-valued wave function and the Born rule, is reinterpreted as a computational convenience rather than a foundational truth. The probability vector and phase vector provide a more transparent, physically grounded description of quantum dynamics, where the Born rule is a byproduct of coordinate transformations rather than an independent axiom.

C. The Phase Network

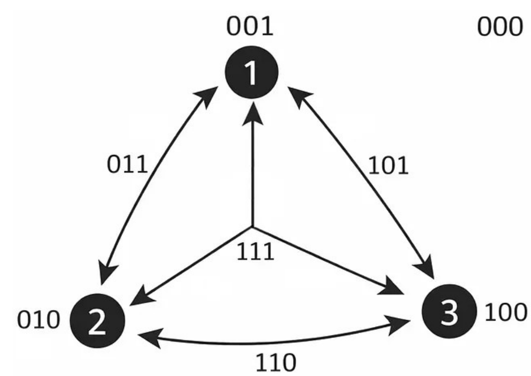
We have argued that the probability vector is epistemic, while the phase vector is ontic. Both vectors are of the same size, as they are both defined over the same configuration space. However, if the phase vector is an ontic state of the system, it cannot be interpreted as a list of possible configurations. Instead, it must be understood as a state of the system itself, rather than a representation of its possible configurations.

At first, this may seem conceptually challenging. A vector that is the same size as a configuration-space vector, such as one that contains 2^N elements for N qubits, does not naturally lend itself to being interpreted as a state of the system. This is because the configuration space of N qubits has 2^N possible states, while the state space of N qubits (i.e., the space of all possible quantum states) is typically described as a 2N-dimensional space. However, in our framework, the phase vector is not a list of possible configurations; it is a state of the system, and as such, it must be interpreted differently from the probability vector.

To clarify, consider the case of three qubits. The configuration space of three qubits has 2^3=8 possible classical states. However, the number of variables needed to describe the state of the system is not 8: it is N = 3, the number of qubits. This raises an important question: how can a vector of size 2^N (like the phase vector) be interpreted as a state of a system with only N degrees of freedom?

The answer lies in interpreting the phase vector not as a list of possible configurations, but as a relational property of the system. Specifically, we conceive of the phase vector as representing a phase network: a structure that encodes how the qubits in the system relate to one another. This is analogous to the structure of an artificial neural network, where nodes, connections, and biases are all represented by parameters.

The first entry of the phase vector (e.g. 000) represents the bias of the network. The entries where only a single bit is 1 (e.g., 001, 010, 100) correspond to nodes in the network. The entries where multiple bits are 1 (e.g., 011, 110, 111) correspond to connections between nodes.



This structure allows the phase vector to encode a relational state of the system, where the phases represent how the qubits are related to one another. The phase network, therefore, is not a physical entity or substance, but rather a relational property of the system, akin to the structure of a neural network.

In this interpretation, the phase vector is not a list of possible configurations but a state of the system, encoding the relationships among its components. This allows it to maintain the same size as the configuration space while still being interpreted as an ontic state of the system. The phase vector's role is not to track configurations but to represent the system's relational structure, which persists independently of the probabilistic uncertainty captured by the probability vector.

The phase network, as a relational property of the system, evolves deterministically throughout the quantum computation. This deterministic evolution stands in contrast to the stochastic behavior of the probability vector, which reflects the observer's epistemic uncertainty about the system's configuration.

While the phase vector changes over time, its evolution follows fixed, deterministic rules, independent of the probabilistic updates that govern the probability vector. Even as the bits' configuration evolves probabilistically, the phase vector’s deterministic character remains intact, preserving its role as a foundational, relational property of the system.

References

[1] J. A. Barandes, The Stochastic-Quantum Correspondence, arXiv:2302.10778 (2023).
[2] S. Kochen and E. P. Specker, J. Math. Mech. 17, 59 (1967).
[3] T. Bayes, Philos. Trans. R. Soc. Lond. 53, 370 (1763).
[4] M. Born, Z. Phys. 37, 863 (1926).